Abstract

In this thesis, new algorithmic methods for ordinary differential equations (ODEs) are developed as well as the foundation of the algebraic basis for these methods. The aim of this thesis is of a rather practical nature, i.e. the presentation of practical and efficient algorithms and heuristics for the solution of ODEs, which can directly be implemented in the framework of a general purpose computer algebra system. The emphasis is clearly put on "practical and efficient". However, in order to achieve this goal, a rather extensive algebraic setup is developed and new and customized notions are designed. Extensions of the methods developed by E. S. Cheb-Terrab et. al. in the 1990s are presented. These extensions can be used to compute integrating factors of third and higher order non–linear ODEs. Furthermore, a new approach for the computation of integrating factors of ODEs arising from the application of special skew symmetric operators is developed. A more general approach and yet unpublished symmetry result by B. Fuchssteiner is presented. New applications of this result are discussed and it is shown, how the symmetry results by Cheb-Terrab et. al. can be used in this more general theoretical setting. Finally, by introduction of a new type of symmetries (non–local symmetries) a link between two theories (differential Galois theory and symmetry analysis), which were up to now considered to be totally disjoint, is established. Technically seen, this amounts to a combination of symmetry methods and the theory of nilpotent flows in order to give a new algorithmic approach for computing the important symmetric powers of linear differential operators.